Fsusy and Field Theoretical Construction

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M.B. SEDRA,[1] and **J. ZEROUAOUI**
Université Ibn Tofail, Faculté des Sciences, Département de Physique,
Laboratoire de Physique de la Matière et Rayonnement (LPMR), Kénitra, Morocco

Abstract

Following our previous work on fractional spin symmetries (FSS) [6, 7], we consider here the construction of field theoretical models that are invariant under the $D = 2(1/3, 1/3)$ supersymmetric algebra.

1 Superspace setup

Fractional supersymmetry [1, 2, 3, 4, 5, 6, 7] is once again considered in this work. The $D = 2(1/3, 1/3)$ superalgebra discussed in [2, 3, 6, 7] is generated by the left and right conserved charges $Q^-_{1/3}$, $Q^+_{1/3}$, P and $\overline{Q}^-_{-1/3}$, $\overline{Q}^+_{-1/3}$, $\overline{P}$ respectively together with four topological charges $\Delta^{(-,-)}$, $\Delta^{(-,+)}$, $\Delta^{(+,-)}$ and $\Delta^{(+,+)}$ relating the two sectors. The $\pm$ and 0 charges carried by these objects are those of the $Z_3 \times \overline{Z}_3$ automorphism symmetry acting as:

$$\begin{aligned}\Gamma Q^+ &= qQ^+,\ \Gamma Q^- = \overline{q}Q^-,\ \Gamma P = P,\\ \overline{\Gamma Q}^+ &= q\overline{Q}^+,\ \overline{\Gamma Q}^- = \overline{q}\overline{Q}^-,\ \overline{\Gamma P} = \overline{P},\\ \overline{\Gamma Q}^\pm &= \overline{Q}^\pm,\ \overline{\Gamma}Q^\pm = Q^\pm,\ \Gamma\overline{P} = \overline{P},\end{aligned} \tag{1}$$

where Γ and $\overline{\Gamma}$ are the generators of the Z_3 and $\overline{Z}_3$ group and where we have used the convention notations $Q^\pm_{-1/3} = \overline{Q}^\pm$ and $P_{-1} = \overline{P}$ in addition to $Q^\pm_{1/3} = Q^\pm$ and $P_1 = P$ used in [7]. The $D = 2(1/3, 1/3)$ supersymmetric algebra admits moreover an extra $Z_2 \times \overline{Z}_2$ symmetry, generated by $C \otimes \overline{C}$, acting as follows:

$$\begin{aligned}CQ^- &= Q^+C,\\ \overline{CQ}^- &= \overline{Q}^+\overline{C}.\end{aligned} \tag{2}$$

Note that under the complex conjugation $(*)$ of complex variables $z^* = \overline{z}$, we have the obvious relations:

$$\begin{aligned}\overline{Q}^+ &= \left(Q^-\right),\\ \overline{P} &= P^*,\end{aligned} \tag{3}$$

[1] Corresponding author: msedra@ictp.it

showing that the left and right sectors are related by the complex conjugation of the two dimensional world sheet parametrized by z and $\overline{z}$. For later use, we quote hereafter the different automorphisms of the $D = 2(1/3, 1/3)$ superalgebra

$$\begin{array}{|c|c|c|c|c|c|c|c|}\hline & Q^+ & Q^- & \overline{Q}^+ & \overline{Q}^- & P & \overline{P} & \Delta^{(-,-)} \\ \hline C & Q^- & Q^+ & \overline{Q}^+ & \overline{Q}^- & P & \overline{P} & \Delta^{(+,-)} \\ \overline{C} & Q^+ & \overline{Q}^- & \overline{Q}^- & \overline{Q}^+ & P & \overline{P} & \Delta^{(-,+)} \\ * & \overline{Q}^- & \overline{Q}^+ & Q^- & Q^+ & \overline{P} & P & \Delta^{(+,+)} \\ \hline\end{array} \quad (4)$$

A differential representation of the $D = 2(1/3, 1/3)$ superalgebra respecting (4) may be obtained by introducing a large superspace (z, $\theta^{\pm}$, $\overline{z}$, $\overline{\theta}^{\pm}$, x^{++}, x^{--}, x^{+-}, x^{-+}) with $\left(\theta^{\pm}\right)^3 = 0$ and $\left(\overline{\theta}^{\pm}\right)^3 = 0$. We find

$$\begin{aligned} D^- &= D^- + \alpha\overline{\theta}^-\partial^{(-,+)} + \alpha\overline{\theta}^+\partial^{(-,-)}, \\ D^+ &= D^+ + \alpha\overline{\theta}^+\partial^{(+,-)} + \alpha\overline{\theta}^-\partial^{(+,+)}, \\ \overline{D}^- &= \overline{D}^- + \overline{\alpha}\theta^-\partial^{(+,-)} + \overline{\alpha}\theta^+\partial^{(-,-)}, \\ \overline{D}^+ &= \overline{D}^+ + \overline{\alpha}\theta^+\partial^{(-,+)} + \overline{\alpha}\theta^-\partial^{(+,+)}, \\ P &= -\overline{q}\frac{\partial}{\partial z}, \overline{P} = -q\frac{\partial}{\partial \overline{z}}, \\ \Delta^{(+,+)} &= (\overline{\alpha} - \alpha q)\partial^{(+,+)}, \Delta^{(-,-)} = (\overline{\alpha} - \alpha\overline{q})\partial^{(-,-)}, \\ \Delta^{(+,-)} &= (\overline{\alpha} - \alpha\overline{q})\partial^{(+,-)}, \Delta^{(-,+)} = (\overline{\alpha} - \alpha q)\partial^{(-,+)}, \end{aligned} \quad (5)$$

where the derivatives along the extra directions, realizing the topological charges as translation generators, are defined as: $\partial^{(+,+)} = \frac{\partial}{\partial x^{--}}$ and so on. $\overline{D}^-$ and $\overline{D}^+$ are the spin $\frac{1}{3}$ charge operators realizing the right sector of the $D = 2(1/3, 1/3)$ superalgebra without topological charges. As shown on the table (4), $\overline{D}^-$ and $\overline{D}^+$ read as :

$$\begin{aligned} \overline{D}^+ &= \partial/\partial\overline{\theta}^- + \overline{\theta}^{-2}\partial/\partial\overline{z} \\ \overline{D}^- &= \partial/\partial\bar{\theta}^+ + \overline{\theta}^{+2}\partial/\partial\overline{z} \end{aligned} \quad (6)$$

Note that $\overline{D}^-$ and $\overline{D}^+$ are related to each other the $\overline{Z}_2$ automorphism group acting on the superspace variables $\theta^{\pm}$ and z as

$$\begin{aligned} \overline{C\theta}^- &= \overline{\theta}^+\overline{C} \\ \overline{C}\overline{z} &= \overline{z}\overline{C} \end{aligned} \quad (7)$$

Superfields describing off shell representations of the $D = 2(1/3, 1/3)$ superalgebra are superfunctions defined on the generalized superspace $(z, \theta, \overline{z}, \overline{\theta}, x)$. They consist of $3^4 = 81$ component fields depending on the bosonic variable z, $\overline{z}$ and x. This is a big number of degrees of freedom that renders very difficult the elaboration of invariant field theoretical models under the $D = 2(1/3, 1/3)$ symmetry. However, forgetting about some automorphism symmetries, one may construct models which are invariant under subalgebras of $D = 2(1/3, 1/3)$. To do that, various possibilities are in order. The simple way is to ignore all the automorphisms given by (4). This is the case of the subalgebra:

$$\begin{aligned} Q^{-3} &= P, \\ \overline{Q}^{-3} &= \overline{P}, \\ Q^-\overline{Q}^- - \overline{q}\overline{Q}^-Q^- &= \Delta^{(-,-)} \end{aligned} \quad (8)$$

generated by non hermitian charge operators. Non unitary invariant models under this symmetry will be considered here. The same thing may be said about the subalgebra generated by $\left(Q^{+}, \overline{Q}^{+}, P, \overline{P}\right)$ as it is related to (8) by the $Z_2 \times \overline{Z}_2$ symmetry generated by $C \otimes \overline{C}$. The second kind of models, which will be studied later, are based on the subalgebra:

$$
\begin{aligned}
\overline{Q}^{-} &= P, \\
\overline{Q}^{+} &= \overline{P}, \\
Q^{-}\overline{Q}^{+} - q\overline{Q}^{+}Q^{-} &= \Delta.
\end{aligned}
\tag{9}
$$

These equations are stable under the complex $(*)$ as shown by (4). The field theory invariant under (9) is real and may describe unitary $D = 2(1/3, 1/3)$ supersymmetric models. In what follows, we first study those theories that are invariant under (8). Introducing the superspace $(z, \theta^{+}, \overline{z}, \overline{\theta}^{+}, x^{++})$ with $\theta^{+3} = 0$ and $\overline{\theta}^{+3} = 0$ a representation of this algebra reads as:

$$
\begin{aligned}
D^{-} &= D^{-} + \alpha\overline{\theta}^{+}\partial^{(-,-)}, \\
\overline{D}^{-} &= \overline{D}^{-} + \beta\theta^{+}\partial^{(-,-)}, \\
P &= -\overline{q}\frac{\partial}{\partial z}, \overline{P} = -q\frac{\partial}{\partial \overline{z}}, \\
\Delta^{(-,-)} &= (\beta - \alpha\overline{q})\partial^{(-,-)},
\end{aligned}
\tag{10}
$$

where D^{-} and $\overline{D}^{-}$ are given by (6). To check that the above relations form indeed a representation of (8), we follow the same strategy as before. First we calculate the the square of $D^{-}(\overline{D}^{-})$. We find:

$$
D^{-2} = D^{-2} + \overline{\theta}^{+2}\partial^{(-,-)2} + (1+\overline{q})\overline{\theta}^{+}\partial^{(-,-)}D^{-},
\tag{11}
$$

where D^{-2} is given by [7]

$$
D^{-2} = \partial^{2}/\partial\theta^{+2} + (1+q)\theta^{+}\partial/\partial\theta^{+}\partial/\partial z + (1+q^{2})\theta^{+2}\partial^{2}/\partial\theta^{+2}\partial/\partial z
\tag{12}
$$

Repeating the some procedure, we get:

$$
D^{-3} = (1+q)\frac{\partial}{\partial z} + \alpha(1+\overline{q}+\overline{q}^{2})[\overline{\theta}^{+}D^{-} + \alpha\overline{\theta}^{+2}\partial^{(-,-)}]D^{-}\partial^{(-,-)},
\tag{13}
$$

which reduces to P because of the identity $1+\overline{q}+\overline{q}^{2} = 0$. A similar proof is valid for $\overline{D}^{-}$. Moreover using the commutation rules:

$$
\begin{aligned}
D^{-}\overline{D}^{-} &= q\overline{D}^{-}D^{-} \\
D^{-}\overline{\theta}^{+} &= \overline{q}\overline{\theta}^{+}D^{-}, \\
\overline{D}^{-}\theta^{+} &= \overline{q}\overline{\theta}^{+}\overline{D}^{-} \\
P\overline{D}^{-} &= \overline{D}^{-}P \\
\Delta^{(-,-)}\overline{D}^{-} &= \overline{D}^{-}\Delta^{(-,-)},
\end{aligned}
\tag{14}
$$

It is not difficult to see that the second equality of (8) is also satisfied.

2 Field theoretical construction

Superfields defined on the superspace $(z, \theta^+, \overline{z}, \overline{\theta}^+, x^{++})$ are usually complex. These off shell representations, consisting of 3^2 complex degrees of freedom, may carry both a spin $s = h - \overline{h}$ and $Z_3 \times \overline{Z}_3$ charge (m, n) with $m, n = 0, \pm 1 (mod 3)$. The $\theta^+_{-1/3}$ and $\overline{\theta}^+_{1/3}$ expansion of a generic superfield $\phi^{(m,n)}_{h,\overline{h}}$ reads as:

$$\begin{aligned}
\phi^{(m,n)}_{h,\overline{h}} &= \varphi^{(m,n)}_{h,\overline{h}} + \theta^+_{-1/3}\psi^{(m-1,n)}_{(h+\frac{1}{3},\overline{h})} + \overline{\theta}^+_{1/3}\eta^{(m,n-1)}_{(h,\overline{h}+\frac{1}{3})} \\
&+ \theta^+_{-1/3}\overline{\theta}^+_{1/3}F^{(m-1,n-1)}_{(h+\frac{1}{3},\overline{h}+\frac{1}{3})} + \theta^{+2}_{-1/3}\chi^{(m-2,n)}_{(h+\frac{2}{3},\overline{h})} \\
&+ \overline{\theta}^{+2}_{1/3}\lambda^{(m,n-2)}_{(h,\overline{h}+\frac{2}{3})} + \theta^{+2}_{-1/3}\overline{\theta}^+_{1/3}\xi^{(m-2,n-1)}_{(h+\frac{2}{3},\overline{h}+\frac{1}{3})} \\
&+ \overline{\theta}^{+2}_{1/3}\theta^+_{-1/3}V^{(m-1,n-2)}_{(h+\frac{1}{3},\overline{h}+\frac{2}{3})} + \theta^{+2}_{-1/3}\overline{\theta}^{+2}_{1/3}D^{(m-2,n-2)}_{(h+\frac{2}{3},\overline{h}+\frac{2}{3})}
\end{aligned} \tag{15}$$

Taking $h = \overline{h} = -1$ and $n = m = -1$, we that the fields $\varphi^{(-,-)}$, $\chi^{(0,-)}_{\frac{2}{3}}$, $\lambda^{(-,0)}_{-\frac{2}{3}}$and $D^{(-,-)}$ are exactly those appearing in the realization of thee critical spin $\frac{1}{3}$ supersymmetry of the TPM namely

$$\begin{aligned}
\varphi^{(-,-)} &= \phi^{(-,-)}_{\frac{1}{21},\frac{1}{21}} \quad , D^{(0,0)} = \phi^{(0,0)}_{\frac{1}{21},\frac{5}{7}} \\
\chi^{(0,-)}_{\frac{2}{3}} &= \phi^{(0,-)}_{\frac{5}{7},\frac{1}{21}} \quad , \lambda^{(0,-)}_{\frac{-2}{3}} = \phi^{(-,0)}_{\frac{1}{21},\frac{5}{7}}
\end{aligned} \tag{16}$$

The remaining fields $\psi^{(-,-)}_{\frac{1}{3}}$, $\eta^{(-,+)}_{-\frac{1}{3}}$, $F^{(+,+)}$, $\xi^{(0,+)}_{\frac{1}{3}}$ and $V^{(+,0)}_{-\frac{1}{3}}$ which are identified with:

$$\begin{aligned}
\psi^{(-,-)}_{\frac{1}{3}} &= \phi^{(+,-)}_{\frac{8}{21},\frac{1}{21}} \quad , \eta^{(-,+)}_{-\frac{1}{3}} = \phi^{(-,+)}_{\frac{1}{21},\frac{8}{21}} \\
\xi^{(0,+)}_{\frac{1}{3}} &= \phi^{(0,+)}_{\frac{5}{7},\frac{8}{21}} \quad , F^{(+,+)} = \phi^{(+,+)}_{\frac{8}{21},\frac{8}{21}} \\
V^{(+,0)}_{-\frac{1}{3}} &= \phi^{(+,0)}_{\frac{8}{21},\frac{5}{7}}
\end{aligned} \tag{17}$$

Are extra conformal fields since they are not predicted by the $C = \frac{6}{7}$ conformal theory. They are however indispensable in the building of a manifestly $D = 2(1/3, 1/3)$ supersymmetric theory eventually invariant under the $Z_3 \times \overline{Z}_3$ discrete symmetry of (8). As for the left sector considered previously, here also the highest θ-component terms of superfields (14) transform as a total space time derivative under the change $\delta\theta^+ = \varepsilon^+$ and $\delta\overline{\theta}^+ = \overline{\varepsilon}^+$. Invariant actions S are then constructed as in $D = 2(1/2, 1/2)$ supersymmetric theories. We have:

$$S = \int d^2z d^4\theta L^{(-,-)}, \tag{18}$$

where $\int d^4\theta \sim \overline{D}^{-2} D^{-2}$ and where the super-Lagrangian $L^{(-,-)}$ carries a $(-,-)Z_3 \times \overline{Z}_3$ charge and scales as $1/3 + 1/3$ dimensional quantity since the integral measure scales as $(\text{length})^{1/3+1/3}$. Note that we have ignored the $x-$dependence realizing the topological charge $\Delta^{(-,-)}$. Other details will given when examining hermitian models. Using dimensional arguments, it is not difficult to see that $L^{(-,-)}$ is of the form:

$$L^{(-,-)} \sim D^-\phi_1^{(m,n)}\overline{D}^-\phi_2^{(-m,-n)} + W^{(-,-)}(\phi_1, \phi_2), \tag{19}$$

where the integers m, n may take the values $0, \pm 1$. As pointed out from the beginning of this work, the action S and the lagrangian (18-19) are not hermitian. Setting $n = m = -1$ as suggested by the thermal deformation of the TPM [6] and using the θ-expansion of the complex superfields $\phi_1^{(-,-)}$ and $\phi_2^{(+,+)}$ as well as the expression of the derivatives D^- and $\overline{D}^-$ and (6), one may calculate the component fields contribution to the action S of the first term of (18). Straightforward algebra leads to:

$$\begin{aligned} D^-\phi_1^{(-,-)} &= \psi_{\frac{1}{3}}^{(+,-)} + \overline{\theta}^+ F^{(+,+)} + q^2\overline{\theta}^{+2} V_{-\frac{1}{3}}^{(+,0)} \\ &\quad -\overline{q}\theta^+\left(\chi_{\frac{2}{3}}^{(0,+)} + \overline{\theta}^+ \xi_{\frac{1}{3}}^{(0,+)} + \overline{\theta}^{+2} D^{(0,0)}\right) \\ &\quad +\theta^{+2}\left(\partial\varphi + \overline{\theta}^+ \partial\eta_{-\frac{1}{3}}^{(-,+)} + \overline{\theta}^{+2}\partial\lambda_{-\frac{2}{3}}^{(-,0)}\right), \end{aligned} \tag{20}$$

$$\begin{aligned} \overline{D}^-\phi_1^{(+,+)} &= \overline{\eta}_{-\frac{1}{3}}^{(+,0)} + \overline{q}\theta^+\overline{F}^{(0,0)} + q\theta^{+2}\overline{\xi}_{\frac{1}{3}}^{(-,0)} \\ &\quad -q\overline{\theta}^+\left(\overline{\lambda}_{-\frac{2}{3}}^{(+,-)} + \theta^+\overline{V}_{-\frac{1}{3}}^{(0,-)} + q\theta^{+2}\overline{D}^{(-,-)}\right) \\ &\quad +\overline{\theta}^{+2}\left(\overline{\partial}\overline{\varphi}^{(+,+)} + \theta^+\overline{\partial\psi}_{\frac{1}{3}}^{(0,-)} + \theta^{+2}\overline{\partial}\overline{\chi}_{\frac{2}{3}}^{(-,+)}\right), \end{aligned} \tag{21}$$

where we have put a bar on the component fields of the superfield $\phi_2^{(+,+)}$ in order to avoid the confusion with the $\phi_1^{(-,-)}$ component fields. Evidently, ∂ and $\overline{\partial}$ mean $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial \overline{z}}$ respectively. Integrating with respect to $d^4\theta$ the superfield kinetic term taking into account the commutation rule $\overline{\theta}^+\theta^+ = q\theta^+\overline{\theta}^+$, we get:

$$\begin{aligned} L_0 \sim\ & \partial\varphi^{(-,-)}\overline{\partial}\overline{\varphi}^{(+,+)} + (\partial\lambda_{-2/3}^{(-,0)}\overline{\eta}_{-1/3}^{(+,0)} - \overline{q}\chi_{2/3}^{(0,-)}\overline{\partial\psi}_{1/3}^{(0,-)}) \\ & - \left(\partial\eta_{-\frac{1}{3}}^{(-,+)}\overline{\lambda}_{-\frac{2}{3}}^{(+,-)} - \overline{q}\psi_{1/3}^{(+,-)}\overline{\partial}\overline{\chi}_{\frac{2}{3}}^{(-,+)}\right) \\ & -\overline{q}\psi - qD^{(0,0)}\overline{F}^{(0,0)} - \overline{q}F^{(+,+)}\overline{D}^{(-,-)} + \overline{q}V_{-1/3}^{(+,0)}\overline{\xi}_{1/3}^{(-,0)} + \overline{q}\xi_{1/3}^{(0,+)}\overline{V}_{-1/3}^{(0,-)}. \end{aligned} \tag{22}$$

Note that this relation contains two kinds of fields. Dynamical fields namely $\varphi^{(-,-)}$, $\psi^{(+,-)}$, $\overline{\eta}^{(-,+)}$, $\lambda^{(-,0)}$ and $\overline{\varphi}^{(+,+)}$, $\overline{\psi}^{(0,-)}$, $\overline{\overline{\eta}}^{(+,0)}$, $\overline{\lambda}^{(+,0)}$. The obey free field equations of motion whose solutions factorise into analytic and antianalytic parts. Auxiliary fields $F^{(+,+)}$, $V^{(+,0)}$, $\xi^{(0,+)}$, $D^{(0,0)}$ and $\overline{F}^{(0,0)}$, $\overline{\xi}^{(-,0)}$, $\overline{V}^{(0,-)}$, $\overline{D}^{(-,-)}$. They appear linearly in L_0 and lead then to constraint equations. Details on the role of these fields will be given later. Note

moreover that (22) is invariant under the following transformations:

$$\begin{aligned}
\delta\varphi^{(m,n)} &= \varepsilon^{+}_{-1/3}\psi^{(m-1,n)}_{1/3} + \overline{\varepsilon}^{+}_{1/3}\eta^{(m,n-1)}_{-1/3} \\
\delta\psi^{(m-1,n)}_{1/3} &= -q\varepsilon^{+}_{-1/3}\chi^{(m-2,n)}_{2/3} + \overline{\varepsilon}^{+}_{1/3}F^{(m-1,n-1)} \\
\delta\eta^{(m,n-1)}_{-1/3} &= -\overline{\varepsilon}^{+}_{1/3}\lambda^{(m,n-2)}_{-2/3} + \overline{q}\varepsilon^{+}_{-1/3}F^{(m-1,n-1)} \\
\delta F^{(m-2,n-1)} &= (1+q)\varepsilon^{+}_{-1/3}\xi^{(m-2,n-1)}_{1/3} - \overline{q\varepsilon}^{+}_{1/3}V^{(m-1,n-2)}_{-1/3} \\
\delta\chi^{(m-2,n)}_{2/3} &= q\varepsilon^{+}_{-1/3}\partial\varphi^{(m,n)} + \overline{\varepsilon}^{+}_{1/3}\xi^{(m-2,n-1)}_{1/3} \\
\delta\lambda^{(m,n-2)}_{-2/3} &= \overline{q\varepsilon}^{+}_{1/3}\overline{\partial}\varphi^{(m,n)} + \varepsilon^{+}_{-1/3}V^{(m-1,n-2)}_{-1/3} \\
\delta\xi^{(m-2,n-1)}_{1/3} &= \overline{q}\varepsilon^{+}_{-1/3}\partial\eta^{(m,n-1)}_{-1/3} - q\overline{\varepsilon}^{+}_{1/3}D^{(m-2,n-2)} \\
\delta V^{(m-1,n-2)}_{-1/3} &= \overline{q\varepsilon}^{+}_{1/3}\overline{\partial}\psi^{(m-1,n)}_{1/3} - q\varepsilon^{+}_{-1/3}D^{(m-2,n-2)} \\
\delta D^{(m-2,n-2)} &= \varepsilon^{+}_{-1/3}\partial\lambda^{(m,n-2)}_{-2/3} - \overline{q\varepsilon}^{+}_{1/3}\overline{\partial}\chi^{(m-2,n)}_{2/3}
\end{aligned} \tag{23}$$

The spin $\pm 4/3$ supersymmetric conserved current $G^{-}$and $\overline{G}^{-}$ generating these transformations are obtained by using the Noether method. They read as:

$$G^{-} = \partial\varphi^{(-,-)}\overline{\psi}^{(0,+)} - q\psi^{(+,0)}\partial\overline{\varphi}^{(+,+)} + \overline{q}\chi^{(0,-)}\overline{\chi}^{(-,+)}, \tag{24}$$

$$\overline{G}^{-} = \overline{q}\overline{\partial}\varphi^{(-,-)}\overline{\eta}^{(+,0)} + \overline{q}\overline{\partial}\overline{\varphi}^{(+,+)}\eta^{(-,+)} + \lambda^{(-,0)}\overline{\lambda}^{(+,-)} \tag{25}$$

Finally, observe that starting from (23, 24) and using the Z_2-symmetries generated by C and $\overline{C}$, we can build the field realisations of the dual current G^{+} and $\overline{G}^{+}$ as follows:

$$G^{+} = CG^{-}C^{-1}, \overline{G}^{+} = \overline{CG}^{-}\overline{C}^{-1}. \tag{26}$$

We have for G^{+} for instance:

$$G^{+} = \partial\varphi^{(+,-)}\psi^{(0,+)} + \psi^{(-,-)}_{1/3}\partial\overline{\varphi}^{(-,+)} + \overline{q}\chi^{(0,-)}_{2/3}\overline{\chi}^{(+,+)}_{2/3} + ..., \tag{27}$$

where $C\varphi^{(+,-)} = \varphi^{(-,-)}C$ and so on. The superpotentiel term $W^{(-,-)}$ is a priori an arbitrary function of the superfields ϕ_1 and ϕ_2 which may be restricted by requiring convariance under the $Z_3 \times \overline{Z}_3$ transformations. The most general form of $W^{(-,-)}$ respecting the $Z_3 \times \overline{Z}_3$ symmetry reads then as

$$\begin{aligned}
W^{(-,-)} &= \sum_m \left(g_m \phi_1^{(-,-)^{3m+1}} + g'_m \phi_2^{(+,+)^{3m+2}} \right) \\
&\quad + \sum_m g''_m \phi_1^{(-,-)^{m+2}} \phi_2^{(+,+)^{m+1}}
\end{aligned} \tag{28}$$

where g_m, g'_m and g''_m are coupling constants. Note that leading linear term in the above equation $g_0\phi_1^{(-,-)}$, integrated with respect to $d^4\theta$, give $g_0 D^{(0,0)}$. It describes exactly the $\phi_{1,3}$ thermal perturbation of the $c = 6/7$ critical theory as shown by (16) and (14). We expect that this term is the mediator of the spontaneous breaking of the $D = 2(1/3.1/3)$ supersymmetry of the TPM. Recall that in the case of the TIM, the $\phi_{1,3}$ field scaling as $3/5 + 3/5$ conformal object breaks spontaneously the $(1/2, 1/2)$ supersymmetry of the

$c = 7/10$ model. Unfortunately, this feature cannot be checked directly on the scalar potential $V(\varphi, \varphi^*)$ since the theory we are considering in this section is non unitary .we shall not pursue this direction. $D = 2(1/3.1/3)$ supersymmetry breaking will be discussed on the following unitary model.

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References

[1] G. W. Semenoff, Phys. Rev. Lett. **61**, 517 (1988);
C. Ahn, D. Bernard, A. LeClair, Nucl Phys. B 346 (1990)409;
D. Bernard and A. LeClair, Nucl Phys. B 340 (1990)721;
A.B. Zamolodchikov, Int. J. Mod. Phys. A4 (1989)4235.

[2] E.H.Saidi, M.B.Sedra, J.Zerouaoui, Class. Quant. Grav. **12** (1995) 1567;

[3] E.H.Saidi, M.B.Sedra, J.Zerouaoui, Class. Quant. Grav. **12**(1995) 2705.

[4] A. Perez, M. Rausch de Traubenberg, P. Simon, Nucl.Phys.B 482(1996)325;

[5] M. Rausch de Traubenberg, P. Simon, Nucl.Phys.B 517(1998)485.

[6] M.B.Sedra, J.Zerouaoui, Adv. Studies Theor. Phys., Vol. 2, 2008, N.20, 965 - 973

[7] M.B.Sedra, J.Zerouaoui, Adv. Studies Theor. Phys., Vol. 3, 2009, no. 7, 273 - 281